Validation of a Low-Order Model for Closed-Loop Flow Control Enable Flight

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A simple low-order model is derived to determine the flow forces and moments on an airfoil that arbitrarily pitches and plunges with the presence of synthetic jet actuation for the use in an adaptive closed-loop control scheme. The low-order model captures the attached flow response of an airfoil in the presence of synthetic jet actuators near the trailing edge. The model includes two explicit non-linear states for fluid variables and can be easily coupled to the rigid body dynamics of the system. The model is validated with high fidelity numerical simulations and experiments. The low-order model agreement with experiments is good for low reduced frequency pitching. The agreement to numerical simulations is also good for reduced frequencies that are an order of magnitude higher than those attainable in experiments.

I. Introduction

The idea of using small, simple active flow control devices that directly affect the flow field over lifting surfaces to create control forces and moments has attracted growing interest over the last decade. Compared to conventional control surfaces, flow control actuators have the potential benefits of reduced structural weight, lower power consumption, higher reliability, and faster output response. Significant work on open-loop flow control has already demonstrated control effectiveness on both static and rigidly moving test platforms. These studies have primarily focused on mitigation of partial or complete flow separation over stalled wing sections or flaps (e.g., Wu et al. (1998), Petz and Wolfgang (2006)). The lift and drag benefits associated with flow attachment enable control in a broader angle-of-attack range, however, these methods provide no direct control as they rely on conventional control surfaces for control actuation, and provide little benefit at moderate flight conditions. A different approach to flow control that emphasizes fluidic modification of the apparent aerodynamic shape of the surface by exploiting the interaction between arrays of surface-mounted synthetic jet actuators and the local cross flow was recently developed (Smith and Glezer (1998), Glezer and Amitay (2002)). With this approach bi-directional pitching moments can be induced by individually controlled miniature, hybrid surface actuators integrated with rectangular, high aspect ratio synthetic jets that are mounted on the pressure and suction surfaces near the trailing edge (DeSalvo and Glezer (2004)). An important attribute of this technique is that it can be effective not only when the baseline flow is separated but also when it is fully attached, namely at low angles of attack such as at cruise conditions.

Despite the amount of the effort devoted to active flow control technology in recent years, a majority of the work published depends on experience and intuition rather than on a fundamental understanding of the flow physics. This is due to the lack of an analytical formulation of the mechanism behind flow actuation. Even though various aspects of flow actuation have been investigated experimentally, mostly on steady
models, it is still difficult to draw conclusions and predict performance on dynamic models. This presents a challenge for feedback controller design as the vast majority of control synthesis techniques are inapplicable.

In our recent studies we demonstrated successful closed-loop control of pitch motion of a 1-DOF wind tunnel model by using the aforementioned actuators with no moving control surfaces (Kutay et. al. (2006) and Muse et. al. (2008)). First a linear controller was designed for the rigid body model of the test model by approximating the actuators as linear static devices. This controller worked well for slow maneuvers where the static actuator assumption holds. For faster maneuvers, requiring higher bandwidth controller design, interactions between the flow and vehicle dynamics get stronger and the linear rigid body model can no longer represent overall system behavior accurately. In these regimes linear controllers that ignore effects of flow actuation have limited performance. A neural network (NN) based adaptive controller was introduced to improve the controller performance by compensating for the modeling errors in the design including the unmodeled dynamics of actuation. We assume that the system dynamics can be written as

\[ \dot{x} = Ax + B(u + \Delta(x, x_f, u)) \]
\[ \dot{x}_f = f(x, x_f, u) \]  

where \( x \) represents the rigid body states of the vehicle, \( x_f \) is the state vector associated with the dynamics of the flow, and \( u = u_{dc} - u_{ad} \) is the control signal to the actuators with \( u_{dc} \) being the output of a linear dynamic compensator and \( u_{ad} \) the adaptive control signal. Matrices \( A \) and \( B \) form the linear system model used to design \( u_{dc} \) and \( \Delta \) represents the modeling errors in rigid body dynamics and the couplings between the vehicle and unmodeled flow dynamics.

For slow maneuvers where changes in the flow field due to actuation occur much faster than the variations in the vehicle states, \( \Delta \) remains comparatively small. In this case the vehicle behavior approaches the linear design model and \( u_{dc} \) alone can control the system sufficiently well as demonstrated in Kutay et. al. (2006). As the vehicle starts moving faster, vehicle-flow interactions get stronger and \( \Delta \) becomes large enough to disturb the vehicles predictable dynamics. The adaptive controller proposed in Kutay et. al. was shown on the experiment to successfully compensate for \( \Delta \) for moderate bandwidths for a 1-DOF airfoil and results with a 2-DOF air foil will be presented in Muse et. al. (2008). The adaptive controller only used \( x \) and \( u \) feedback to compensate for \( \Delta \). This is possible only if the \( x_f \) dependence of \( \Delta \) is observable from \( x \) and \( u \), which was evidently the case. For more aggressive maneuvers the observability assumption is likely to fail, or the dependence becomes much more complex. For such cases feedback from flow states is necessary to control the vehicle, which requires some information on \( f(x, x_f, u) \) and \( \Delta(x, x_f, u) \).

The objective of this research is to develop a low-order approximate flow model for the wind tunnel setup at Georgia Tech that has been extended to 2-DOF by the addition of the plunge axis, validate it with the experiment data, and utilize it for simulation and controller design. For our adaptive controller design the required modeling information is minimal and can possibly be obtained through testing. By using only the relative degree of regulated variable (vehicle position) with respect to \( u \), the adaptive controller can theoretically compensate for \( \Delta \) for a completely unknown \( f \). If some information on \( f \) is available, this can be incorporated into the linear part of the design (\( A \) and \( B \) matrices in Eq. (1)), effectively shifting a part of flow control from the adaptive controller to the linear controller. In this regard it is desirable to have a mathematical model for \( f(x, x_f, u) \) and \( \Delta(x, x_f, u) \) so that the adaptive controller will need to learn a smaller portion of the uncertainty with potentially superior results. The low-order model will also be used to simulate the system with higher accuracy. This is crucial to allow for investigation of various controller architectures without spending expensive wind tunnel time.

II. Experimental Setup and the Modeling Problem

The low-order model is developed to describe the problem of an airfoil in free flight. This problem has been realized in an experimental apparatus designed at the Georgia Institute of Technology, Atlanta. The apparatus is thus used to provide detailed validation for the low-order model and will become the testbed for future control investigations. To describe the modeling problem we now discuss the details of the experimental setup.
The experiments are conducted in an open-return low-speed wind tunnel. The present experiments use a 2-D airfoil model with a fixed cross section that is based on a NACA 4415 configuration as shown in figure 1a. The chord length is $c = 457$ mm, maximum thickness to chord ratio is $t/c = 0.15$, and the model spans the entire width of the wind tunnel test section. Bi-directional pitching moments induced by trapped vorticity flow-control are created by individually-controlled miniature, hybrid surface actuators integrated with rectangular, high aspect ratio synthetic jets that are mounted on the pressure and suction surfaces near the trailing edge (figure 1a). The actuators have a characteristic height of $0.017c$ above the airfoil surface and the long dimension of the exit plane of each rectangular jet is parallel to the trailing edge with the height (in the cross stream direction) of each jet orifice is 0.4 mm. The jets are generated by piezoelectric membranes that are built into a central cavity within the actuator and are operated off resonance within the range $1770\,\text{Hz} < f_{\text{act}} < 2350\,\text{Hz}$. The spanwise-segmented actuators are individually controlled from the laboratory computer and the system controller. The bulk of the present experiments are conducted at a free stream speed of $U_\infty = 30\,\text{m/s}$, with a corresponding Reynolds number based on the airfoil chord length of $Re_c = 8.55 \times 10^5$. At this speed, the actuation Strouhal number is $St = f_{\text{act}}c/U_\infty = 34$ and the maximum momentum coefficient is $C_\mu = 1 \cdot 10^{-3}$.

In the present experiments, the wind tunnel model is able to execute commanded flight maneuvers in two degrees of freedom (pitch and plunge). Although we are currently investigating 2-DOF motion, the model is mounted on a programmable, 3-DOF (pitch, plunge, and roll) traverse that is constructed on an I-beam frame around the test section of the wind tunnel as shown in figure 1b. The traverse is driven electromechanically by a dedicated feedback controller that removes the effect of parasitic mass and rotational inertia of the dynamic support system. Thus the experimental apparatus is able to mimic the airfoil in free-flight.

### III. Aerodynamic Model

A low-order model for a pitching and plunging airfoil with trailing-edge flow control is presented in this section. Unlike other low or reduced-order models, this model is built from physical principles, i.e. the conservation of momentum. The vorticity in the model consists of freely-moving vortices in the wake, a trapped control vortex with circulation $\Gamma_C(t)$, and the boundary-layer vorticity on the airfoil surface. We consider small-amplitude motions of a flat-plate airfoil which leads to a number of simplifying assumptions. Thus, for example, all but one of the wake vortices move uniformly downstream along the nominal $x$-axis that is fixed to the airfoil. The exception is the vortex being fed circulation from the trailing edge. The velocity of the latter vortex is modified to conserve momentum as discussed below. In addition we assume that vorticity is shed into the wake to satisfy the unsteady Kutta condition (no velocity singularity) at the trailing edge. Also, a vortex sheet with a continuous distribution of vorticity $\gamma(x)$ is used to satisfy the boundary condition.
on the airfoil surface (normal component of velocity is continuous) plus the Kutta condition.

For the control input, the synthetic jet is modeled with a trapped vortex that represents the averaged effect of the actuation. The control vortex circulation depends on the control variable, \( u \), for example,

\[
\frac{d\Gamma_C}{dt} = F(u, \Gamma_C).
\]

This function, \( F \), can only be experimentally determined.

**III.A. Wake Vortex Dynamics**

Consider an airfoil of chord length \( c \) occupying the portion \(-c/2 \leq x \leq c/2\) of the \( x\)-axis as presented in figure 2. The airfoil is undergoing small-amplitude motions in pitch and plunge. The pivot point for rotation is located at \( x = -a \) as shown. The airfoil velocity is \( U \) in the negative \( x\)-direction. Alternatively, one could apply the results below to a stationary airfoil with a freestream velocity \( U \) in the positive \( x\)-direction. In figure 2 the fluid flow and the airfoil motion are given relative to a frame of reference moving in the negative \( x\)-direction with the airfoil.

![Figure 2. Pitching, plunging airfoil, control vortex with strength \( \Gamma_C \), and free wake vortices with strengths \( \Gamma_i \), \( i = 1, ..., N \).](image)

Now also consider a wake vortex with circulation \( \Gamma_i \) located at \( x = \xi_i \). This vortex will result in a given distribution of vorticity on the airfoil, \( \gamma_i(x) \), to satisfy the boundary condition on the airfoil and the Kutta condition given by (see, e.g., von Karman and Sears (1938))

\[
\gamma_i(x) = \frac{\Gamma_i}{\pi(\xi_i - x)} \sqrt{\frac{c/2 - x}{c/2 + x}} \sqrt{\frac{\xi_i + c/2}{\xi_i - c/2}}.
\]

We find that

\[
\int_{-c/2}^{c/2} \gamma_i(x) dx = \Gamma_i \left[ \sqrt{\frac{\xi_i + c/2}{\xi_i - c/2}} - 1 \right].
\]

A control vortex is introduced to represent the effect of actuation. It appears that the correct location of the control vortex should be slightly forward of the trailing edge and just above the airfoil for suction-side actuation and just below the airfoil for pressure-side actuation. The total effect of both actuators can be represented by a single vortex on the mid-plane located slightly forward of the trailing edge. In any event, the control vortex with strength \( \Gamma_C \) will produce a corresponding contribution to the airfoil circulation given
by $\gamma_C(x)$. Based on arguments given in section III.E, we assume that a change in $\Gamma_C$ on its own produces no net change of circulation in the wake. Thus,

$$\int_{-c/2}^{c/2} \gamma_C(x) dx = -\Gamma_C. \quad (4)$$

From now on, we assume that the control vortex is located forward of the trailing edge (and possibly displaced vertically from the airfoil) so that

$$\int_{-c/2}^{c/2} \gamma_C(x) x dx = -\Gamma_C \xi_C. \quad (5)$$

The total circulation, which must equal 0, is therefore given by

$$\Gamma_0 + \sum_{i=1}^{N} \Gamma_i \sqrt{\frac{\xi_i + c/2}{\xi_i - c/2}} = 0.$$

Here $\Gamma_0$ is the quasisteady circulation about the airfoil that depends only on the pitch angle, its time derivative, and the plunge rate, and $N$ is the number of free vortices in the wake.

It is also assumed that all but one of the free vortices in the wake move with speed $U$. Thus,

$$\frac{d\xi_i}{dt} = U \quad (i \geq 2), \quad (6)$$

and the vortex being fed circulation (labelled $i = 1$) moves with speed

$$\frac{d\xi_1}{dt} = U - \frac{(\xi_1^2 - c^2/4)}{\xi_1 \Gamma_1} \frac{d\Gamma_1}{dt} \quad (7)$$

where we have used conservation of impulse (see Eq. (10) below) to derive Eq. (7). This is in contrast with previous models where the force on the vortex and branch cut system is forced to be invariant.\textsuperscript{10–13} It has been found that this so-called Brown-Michael correction introduces the incorrect initial lift curve for the specific case of the flat-plate undergoing an impulsive start. The conservation of impulse argument given above, correctly captures the initial behavior for this specific case.

Except for the vortex being fed, all vortices in the wake remain at constant circulation, i.e.

$$\frac{d\Gamma_i}{dt} = 0 \quad (i \geq 2). \quad (8)$$

The strength of $\Gamma_1$ is such that Eq. (5) remains satisfied. As mentioned earlier, the circulation of the control vortex is given, for example, by

$$\frac{d\Gamma_C}{dt} = F(u, \Gamma_C), \quad (9)$$

where $u$ is the control variable and the function $F$ is to be determined. With the absence of a model for Eq. (9), Eqs. (5)-(8) constitute a close system of equations for the wake dynamics of the system given a sufficient initial condition. Since Eqs. (6) and (8) are easily integrable, this system can be reduced to one non-linear differential, i.e. Eq. (7), with the algebraic constraint represented by Eq. (5). The non-linearity of creating new vortices will be discussed section III.D. In accordance to Eq. (1), the fluid states are represented by $x_f = [\xi_1, \Gamma_1]^T$. In contrast to other models, the fluid state is directly related to the physical variables of vortex location and strength.

### III.B. Lift and Moment

The forces and moments on the airfoil can be obtained from the fluid impulse.\textsuperscript{14} The total fluid impulse in the $y$-direction, $I_y$, is given by

$$I_y = -\rho \left[ \int_{-c/2}^{c/2} \gamma(x) x dx + \sum_{i=1}^{N} \Gamma_i \xi_i + \Gamma_C \xi_C \right]. \quad (10)$$
where $\gamma(x)$ is the total distribution of circulation about the airfoil, consisting of $\gamma_0(x)$ (corresponding to $\Gamma_0$), $\gamma_C(x)$, and the sum of the $\gamma_i(x)$. Using Eq. (3), Eq. (10) becomes

$$I_y = -\rho \left[ \int_{-c/2}^{c/2} \gamma_0(x) + \gamma_C(x) \right] dx + \sum_{i=1}^N \Gamma_i \left[ \xi_i^2 - c^2/4 + \Gamma_C \xi_C \right].$$

(11)

The terms involving $\Gamma_C$ and $\gamma_C$ in Eq. (11) are left separated because their time derivatives are different. The lift, $L$, on the airfoil is given by

$$L = -\frac{dI_y}{dt}.$$  (12)

Using Eqs. (4)-(8) and (11)-(12), we find that the lift is given by

$$L = \rho \frac{d}{dt} \int_{-c/2}^{c/2} \gamma_0(x) dx - \rho U \Gamma_0 - \frac{\rho U c}{2} \sum_{i=1}^N \frac{\Gamma_i}{\sqrt{\xi_i^2 - c^2/4}} + \rho U \Gamma_C.$$  (13)

The first term on the right hand side above is the so-called “added mass” term due to airfoil acceleration, the second term is the quasisteady lift, and the remaining terms represent the lift due to vortices in the near wake and the lift due to the control vortex. In the following section we will determine the first two contributions above and the analogous contributions in the moment equation (see Eq. (15) below) in terms of pitch angle, its time derivative, and the plunge rate.

The total moment of fluid momentum, $M(s)$, at a distance $s$ upstream of the midpoint of the airfoil is given by

$$M(s) = -\frac{1}{2} \rho \left[ \int_{-c/2}^{c/2} \gamma(x) (x + s)^2 dx + \sum_{i=1}^N \Gamma_i (s + \xi_i)^2 + \Gamma_C (s + \xi_C)^2 \right].$$

The moment on the airfoil $M(a)$ about a pivot point a distance $a$ upstream of midchord is then given by $-dM(s)/dt$ evaluated at $s = a$, where one must take into account that $ds/dt = -U$. We find that

$$M(a) = \rho \frac{d}{dt} \left[ \int_{-1}^1 \gamma(x) x^2 dx + \sum_{i=1}^N \Gamma_i \xi_i^2 + \Gamma_C \xi_C^2 \right] + UI_y + aL.$$  (14)

After considerable substitutions and manipulations, using Eqs. (3), (4)-(8), (11), and (14) we can write express $M(a)$ as

$$M(a) = aL + \rho \frac{d}{dt} \int_{-c/2}^{c/2} \gamma_0(x) (x^2 - c^2/8) dx$$

$$-\rho U \int_{-c/2}^{c/2} \gamma_0(x) dx + \frac{\rho U c^2}{8} \sum_{i=1}^N \frac{\Gamma_i}{\sqrt{\xi_i^2 - c^2/4}} + \rho U \Gamma_C \xi_C$$

$$-\rho \left[ \frac{(\xi_1^2 - c^2/4)^{3/2} d\Gamma_1}{\xi_1} \right].$$  (15)

We recommend neglecting the last term with the expression above for $M(a)$ to agree with the classical result that the lift force due to wake vorticity acts at the quarter-chord point on the airfoil. Recall that Eq. (7) for the velocity $d\xi_1/dt$ is derived using conservation of impulse. However, moment of momentum is not necessarily conserved and neglecting this last term corrects this oversight.

III.C. Added Mass and Quasisteady Lift and Moment

Airfoil translation and rotation leads to quasisteady bound circulation about the airfoil which we denote by $\gamma_0(x)$ and $\gamma_C(x)$, respectively, with their sum equal to $\gamma(x)$. A nonzero vertical velocity of the airfoil $v_y$ measured at the midchord must be matched by an equal fluid velocity, constant along the chord. Thus, we require
\[ \frac{1}{2\pi} \int_{-1}^{1} \frac{\gamma_0(s)ds}{x-s} + U\theta = v_y = \dot{y} - a\dot{\theta} \quad (-c/2 \leq x \leq c/2), \]

where the integral is taken using principal-value integration. We find that \( \gamma_0(x) \), satisfying the Kutta condition, is given by

\[ \gamma_0(x) = 2(\dot{y} - a\dot{\theta} - U\theta) \sqrt{\frac{c/2 - x}{c/2 + x}} \]  

(16)

from which we obtain

\[ \Gamma_0t = \int_{-c/2}^{c/2} \gamma_0(x)dx = \pi c(\dot{y} - a\dot{\theta} - U\theta), \]

\[ \int_{-c/2}^{c/2} \gamma_0(x)x dx = -\frac{c}{4} \Gamma_0t, \]

\[ \int_{-c/2}^{c/2} \gamma_0(x)x^2 dx = \frac{c^2}{8} \Gamma_0t. \]

Similarly, airfoil rotation about the midchord produces an airfoil vertical velocity distribution of \( -\dot{\theta}x \) to be matched by a velocity distribution induced by the circulation distribution \( \gamma_r(x) \). Thus, we require

\[ \frac{1}{2\pi} \int_{-c/2}^{c/2} \frac{\gamma_r(s)ds}{x-s} = -\dot{\theta}x \quad (-c/2 \leq x \leq c/2). \]  

(17)

The solution to Eq. (17), satisfying the Kutta condition, is

\[ \gamma_r(x) = -2\dot{\theta} \sqrt{c^2/4 - x^2} \]  

(18)

from which we obtain

\[ \Gamma_0r = \int_{-c/2}^{c/2} \gamma_r(x)dx = -\frac{\pi c^2 \dot{\theta}}{4}, \]

\[ \int_{-c/2}^{c/2} \gamma_r(x)x dx = 0, \]

\[ \int_{-c/2}^{c/2} \gamma_r(x)x^2 dx = \frac{c^2}{16} \Gamma_0r. \]

Using the above results for \( \Gamma_0t \) and \( \Gamma_0r \), we see that

\[ \Gamma_0 = \Gamma_0t + \Gamma_0r = \pi c \left[ \dot{y} - (a + \frac{c}{4})\dot{\theta} - U\theta \right]. \]

Collecting the results of this section and applying them to the expressions for lift and moment, Eqs. (13) and (15) respectively, we find

\[ L = -\rho \pi \left( \frac{c^2}{4} \dot{y} + Uc\dot{y} \right) + \rho \pi \left[ \frac{ac^2}{4} \dot{\theta} + U(a + \frac{c}{2})c\dot{\theta} + \left( \frac{Uc^2}{4} + U^2 c \right) \theta \right] \]

\[ -\frac{\rho Uc}{2} \sum_{i=1}^{N} \frac{\Gamma_i}{\sqrt{\xi_i^2 - c^2/4}} + \rho U \Gamma_C, \]  

(19)
\[ M(a) = aL + \frac{\rho \pi U c^2}{4} \dot{y} + \rho \pi \left[ \frac{c^4}{128} \ddot{\theta} - \frac{U a c^2}{4} \dot{\theta} - \frac{U^2 c^2}{4} \theta \right] \]
\[ + \frac{\rho U c^2}{8} \sum_{i=1}^{N} \frac{\Gamma_i}{\sqrt{\xi_i^2 - c^2/4}} + \rho U \Gamma c \xi_c, \]  

(20)

where we have neglected the last term in Eq. (15) for the reason stated above. The lift coefficient, defined as \( C_L = 2L/(\rho U c^2) \), is

\[ C_L = -\pi \left( \frac{c}{2U^2} \ddot{y} + \frac{2}{U} \dot{y} \right) + \pi \left[ \frac{ac}{2U^2} \ddot{\theta} + \frac{2a + c}{U} \dot{\theta} + \frac{c\dot{U} + 4U^2}{2U^2} \theta \right] \]
\[ - \frac{1}{U} \sum_{i=1}^{N} \frac{\Gamma_i}{\sqrt{\xi_i^2 - c^2/4}} + \frac{2}{Uc} \Gamma C, \]

(21)

and the moment coefficient (with pitch up as positive) \( C_M = -2M(a)/(\rho U^2 c^2) \) is

\[ C_M = -\frac{a}{c} C_L - \frac{\pi}{2U^2} \ddot{y} - \pi \left[ \frac{c^2}{64U^2} \ddot{\theta} - \frac{a}{2U} \dot{\theta} - \frac{1}{2} \right] \]
\[ - \frac{1}{4U} \sum_{i=1}^{N} \frac{\Gamma_i}{\sqrt{\xi_i^2 - c^2/4}} - \frac{2\xi_C}{Uc^2} \Gamma C. \]

(22)

### III.D. Integration in Time

Equation (7) is singular when \( t = 0 \). In general, a small-time, asymptotic solution is necessary to provide a sufficient initial condition for the startup of the simulation. In this section we outline an alternative strategy to determine \( \xi_1(t) \) and \( \Gamma_1(t) \).

From the conservation of total circulation, Eq. (5), we can write

\[ \Gamma_1 \sqrt{\xi_1 + c/2} = \Gamma_0 - \sum_{i=2}^{N} \Gamma_i \sqrt{\xi_i + c/2} = G(t), \]

(23)

where the newly defined function \( G(t) \) can be considered known up to time \( t \). In addition, Eq. (7) can be written as

\[ \frac{d}{dt} \left( \sqrt{\xi_1^2 - c^2/4} \Gamma_1 \right) = \frac{\xi_1 \Gamma_1 U}{\sqrt{\xi_1^2 - c^2/4}} = \frac{\xi_1 GU}{\xi_1 + c/2}. \]

(24)

Defining

\[ H(t) = \sqrt{\xi_1^2 - c^2/4} \Gamma_1, \]

(25)

we can write Eq. (24) as

\[ \frac{dH}{dt} = \frac{H + cG/2}{H + cG} GU, \]

(26)

where we have used Eqs. (23) and (25) to determine \( \xi_1 \) as

\[ \xi_1 = \frac{H + cG/2}{G}. \]

Similarly, \( \Gamma_1 \) is found to be
\[ \Gamma_1 = G \sqrt{\frac{H}{H + cG}}. \]

Integrating Eq. (26) numerically is now straightforward. Physically we expect that \( \Gamma_1(t) \) cannot decrease in magnitude as time progresses. Let \( t^* \) be a time when \( d\Gamma_1/dt \) changes sign. Thus we check, after each time increment, to see if \( \Gamma_1 \) has decreased in magnitude. If so, we return to the previous time \( t \), which, by definition, \( t = t^* \), and add the present \( \Gamma_1 \) and \( \xi_1 \) to the list of wake vortices labelled \( i \geq 2 \) and form a new \( i = 1 \) vortex with initial conditions \( \xi_1(t^*) = c/2 \) and \( \Gamma_1(t^*) = 0 \). Typically, near \( t \approx t^* (t > t^*) \) we expect \( G \sim t - t^* \) which leads to \( \xi_1 \approx 1 + Ut(t - t^*)/4 \) and \( \Gamma_1 \approx G(t)\sqrt{Ut(t - t^*)}/8 \) for \( U(t(t - t^*)) \ll 1 \). An exception to this behavior would occur, for example, for an impulsive start at \( t = 0 \) with say \( G(0) = -\Gamma_0(0) \neq 0 \). In this case, \( \xi_1 \approx 1 + Ut/2 \) and \( \Gamma_1 \approx -\Gamma_0(0)\sqrt{Ut}/4 \) for \( U(t - t^*) \ll 1 \).

### III.E. Control Vortex

It has been previously shown that the usage of synthetic jets traps vorticity in the boundary layer to directly modify the flow in the average sense.\(^4,15\) This can be also be viewed as “virtual shaping” of the airfoil since this modifies the streamlines in the vicinity of the actuator. Here we model this “virtual shaping” or trapped vorticity as a stationary control vortex that depends on the control parameter (e.g. voltage), \( u \). We have used DeSalvo and Glezer (2007) as a guide in developing a model for the control vortex. Figure 5 of DeSalvo and Glezer (2007) shows that the lift force acts near the actuation point, at least for the case of actuation at 95% chord as shown and thus we assume that the effect of actuation is “local.” In fact, the \( \Delta \rho_c \) observed is consistent with a control vortex at 95% chord or \( \xi_C = 0.45c \) and slightly above the airfoil for suction-side (SS) actuation and slightly below the airfoil for pressure-side (PS) actuation. Since the PS and SS actuation do not act at the same time, the control vortex can be thought of as one vortex (situated on the x-axis) with a positive strength representing PS actuation and a negative strength representing SS actuation. Note also from figure 5 of DeSalvo and Glezer (2007) that the pressure distribution away from the neighborhood of the actuation point is essentially unaffected. This leads to the conclusion that no net circulation is injected into the wake as a direct consequence of turning the actuation on or off.

We can also define the vertical distance of the control vortex from the airfoil as \( \eta_C \) to introduce another added degree of freedom to the model and address the sensing problem. If sufficiently close to the airfoil, this quantity is not required for the force or moment but is needed to determine the pressure distribution due to actuation. This quantity may be estimated as follows. A point vortex, stationary with respect to the airfoil, with strength \( \Gamma_C \) and located at \( (\xi_C, \eta_C) \) relative to the airfoil will produce an incremental tangential velocity along the actuation side of the airfoil surface of\(^a\)

\[
\Delta u_C(x) = \frac{\eta_C \Gamma_C}{\pi [(x - \xi_C)^2 + \eta_C^2]}. \tag{27}
\]

For SS actuation we have \( \gamma_{C,SS} = -\Delta u_C(x) \) and for PS actuation \( \gamma_{C,PS} = +\Delta u_C(x) \). The resulting pressure change acts only on the actuation side and is given by

\[
\Delta p_C(x) = -\frac{\rho U \eta_C \Gamma_C}{\pi [(x - \xi_C)^2 + \eta_C^2]},
\]

and the corresponding change in pressure coefficient is

\[
\Delta C_{p,C}(x) = -\frac{2\eta_C \Gamma_C}{\pi U [(x - \xi_C)^2 + \eta_C^2]}.
\]

Referring to figure 5 of DeSalvo and Glezer (2007), we note the choice \( |\eta_C| \approx 0.025c \), close to the characteristic height of the actuators of 0.017c in DeSalvo and Glezer (2007), would produce a reasonable fit of Eq. (27) to their data presented in figure 5.

\(^a\)u in this case is the standard velocity in the x-direction. It will be made obvious when this definition of \( u \) is used as opposed to the control input.
For simplicity, we choose to investigate more closely the case where we model the actuation with a single vortex located on the x-axis. If the timescale for formation of $\Gamma_C << c/U$ then it can be assumed that the control vortex depends on only the control input, $u$, and the angle of attack, $\theta$, such that

$$\Gamma_C = g(u, \theta).$$

Given this assumption, one can generate a lookup table (or approximate function) for $\Gamma_C$ based on steady state information. If there are no dynamics in the system we find

$$C_M(u, \theta) = \frac{\pi \theta}{2} \left( 1 - \frac{4\alpha}{c} \right) - \frac{2\Gamma_C}{Uc^2} (a + \xi_C).$$

Subtracting the moment at $u = 0$ we obtain

$$\Delta C_M(u, \theta) = C_M - C_{M,u=0} = -\frac{2\Gamma_C}{Uc^2} (a + \xi_C).$$

Now an equation for $\Gamma_C$ given the distance $\xi_C$ (e.g. $\xi_C = 0.45c$) is given by

$$\frac{\Gamma_C}{Uc} = -\frac{1}{2} \left( \frac{a}{c} + \frac{\xi_C}{c} \right)^{-1} \Delta C_M.$$ (28)

Thus we conduct the following: we span the space of $u \in [-1, 1]$ and $\theta \in [\theta_{min}, \theta_{max}]$ and calculate the steady moment (after transients have been removed), $C_M$. Next, Eq. (28) is used to generate a look up table, i.e. interpolation is used to find the value of $\Gamma_C$ given $u$ and $\theta$. Plotted in figure 3 is the approximate function corresponding to Eq. (28). Notice that the function is nearly independent of angle of attack as expected. The trapped vorticity should not depend strongly on angle of attack but instead should depend strongly on the voltage supplied to the actuators.

Although this is sufficient for the situations where the time-scale for the formation of the vortex is much smaller than the convective time-scale, a more rigorous method of identification of a function in the form of Eq. (2) may be be needed.

**III.F. Pressure Distribution**

For the sensing problem, the pressure distribution of the airfoil must also be obtained. The pressure distribution, $p(x,t)$, can be obtained by integrating in $x$ the linearized Euler equation
\[
\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left( \frac{p}{\rho} + U u \right) = 0
\]

along the upper or lower airfoil surface. With the exception of the airfoil circulation due to control, \( \gamma_C(x) \), the perturbation velocity \( u \) is given by \(-\gamma(x)/2\) along the upper surface and \(+\gamma(x)/2\) along the lower surface. The perturbation velocity due to control is given by Eq. (27) and is applied on the actuation side of the airfoil only. Thus we have

\[
p(x, t) = \pm \frac{\rho U \tilde{\gamma}(x)}{2} + \frac{\rho}{2} \frac{d}{dt} \int_{-c/2}^{x} \tilde{\gamma}(s) ds + p_{\infty}
\]

where the upper (\(+\)) sign refers to the upper surface, the lower (\(-\)) sign to the lower surface, and \( \tilde{\gamma}(x) \) is the circulation distribution given by

\[
\tilde{\gamma}(x) = \gamma_0 t(x) + \gamma_0 r(x) + \sum_{i=1}^{N} \gamma_i(x) + 2 I_{act} \gamma_C(x)
\]

(29)

where \( I_{act} = 1 \) for the actuation-side pressure and \( I_{act} = 0 \) otherwise.

Equations (16) and (18) give \( \gamma_0 t(x) \) and \( \gamma_0 r(x) \), respectively. The following corresponding integrals may be used in Eq. (29) to give

\[
\int_{-c/2}^{x} \sqrt{\frac{c/2 - s}{c/2 + s}} ds = \sqrt{\frac{c}{4} - x^2} + \frac{c}{2} \left( \sin^{-1} \left( \frac{2x}{c} \right) + \frac{\pi}{2} \right)
\]

and

\[
\int_{-c/2}^{x} \sqrt{\frac{c^2/4 - s^2}{c^2/4}} ds = \frac{1}{2} \left[ x \sqrt{\frac{c^2}{4} - x^2} + \frac{c^2}{4} \left( \sin^{-1} \left( \frac{2x}{c} \right) + \frac{\pi}{2} \right) \right].
\]

The quantity \( \gamma_i(x) \) is given by Eq. (3). Here the integral

\[
\sqrt{\frac{\xi_i + c/2}{\xi_i - c/2}} \int_{-c/2}^{x} \sqrt{\frac{c/2 - s}{c/2 + s}} \left( \frac{1}{\pi(\xi_i - s)} \right) ds = - \left[ \frac{1}{2} - \frac{1}{\pi} \sin^{-1} \left( \frac{c/2 - 2\xi_i x}{\xi_i - x} \right) \right]
\]

\[
- \sqrt{\frac{\xi_i + c/2}{\xi_i - c/2}} \left[ \frac{1}{\pi} \sin^{-1} \left( \frac{2x}{c} \right) + \frac{1}{2} \right]
\]

will be useful. The quantity \( \gamma_C(x) \) can be gotten from the velocity difference between the upper and lower surface due to the vorticity of the control vortex, \( \Gamma_C \), in a similar fashion. If a model for \( \Gamma_C \) is determined i.e. from section III.E, then \( \gamma_C \) can be easily found in terms of the arctangent function through integration.

### III.G. Rigid-Body Dynamics

Dealing with the fluid and body interaction problem is usually a difficult problem. For this case, due to the low-order fluid model, a closed system of equations can be constructed to predict the orientation and the motion of the airfoil. As an pedagogical example, the airfoil is attached to a spring and damper in the \( y \)-direction in addition to a torsion spring and damper in the \( \theta \)-direction. This stabilizes the system and gives a model for non-stalled flutter in cross-flow.

The system of equations for the airfoil in this configuration becomes

\[
\begin{align*}
\dot{m} \ddot{y} - S_x \ddot{\theta} + b_y \dot{y} + k_y y &= L \\
I \ddot{\theta} - S_x \ddot{y} + b_y \dot{\theta} + k_y \theta &= M(a)
\end{align*}
\]

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where $L$ is the lift, $M(a)$ is the moment about the location $a$, $S_x$ is the static imbalance per unit width, and all other terms are related to the linear and rotational mass, damping, and stiffness of the system. The static imbalance per unit width is defined as

$$S_x \equiv \int \xi \rho_s d\xi d\eta.$$  

This is an area integral in a principle coordinate system where $\rho_s$ is the density of the structure, and $\xi$ and $\eta$ are the principle coordinates. This can equivalently be expressed as

$$S_x = ma$$

where $m$ is the mass of the object and $a$ is the distance from the elastic axis (the point at which the springs and dampers are attached) to the center of mass, previously defined in figure 2. When the above equations are combined with Eqs. (19) and (20) we obtain

$$\left[ m + \rho \pi \frac{c^2}{4} \right] \ddot{y} + \left[ \rho \pi \frac{ac^2}{4} - S_x \right] \dot{\theta} = -[b_y + \rho \pi Uc] \dot{y} - k_y y$$

and

$$-\frac{\rho Uc}{2} \left( \frac{\Gamma_1}{\sqrt{\xi^2 - \frac{c^2}{4}}} + \sum_{i=2}^{N} \frac{\Gamma_i}{\sqrt{\xi^2_i - \frac{c^2}{4}}} \right) + \rho UT C$$

for the translational motion in the $y$-direction and

$$\left[ a \rho \pi \frac{c^2}{4} - S_x \right] \ddot{y} + \left[ I + \rho \pi \left( \frac{a^2 c^2}{4} + \frac{c^4}{128} \right) \right] \dot{\theta} = -[b_\theta + a \rho \pi Uc \left( a + \frac{c}{4} \right)] \dot{\theta}$$

and

$$-\left[ k_\theta + \rho \pi U^2 c \left( a - \frac{c}{4} \right) \right] \theta$$

and

$$-\left[ \rho Uc \left( a - \frac{c}{4} \right) \right] \dot{y}$$

and

$$-\left[ \rho Uc \left( a - \frac{c}{4} \right) \right] \left( \frac{\Gamma_1}{\sqrt{\xi^2 - \frac{c^2}{4}}} + \sum_{i=2}^{N} \frac{\Gamma_i}{\sqrt{\xi^2_i - \frac{c^2}{4}}} \right) + \rho UT C \xi C$$

for the rotational motion. We assume $U$ is constant. When a vortex is initially shed, the two equations are singular because $\xi C \to c/2$. To rectify this one can substitute, from Eq. (23),

$$\frac{\Gamma_1}{\sqrt{\xi^2 - \frac{c^2}{4}}} = -\frac{1}{\xi_1 + \frac{c}{2}} \left( \Gamma_0 + \sum_{i=2}^{N} \Gamma_i \left( \frac{\xi_1 + \frac{c}{2}}{\xi_1} \right) \right)$$

Note that a right hand system has been used in this case, thus positive $\theta$ (or moment) corresponds to pitch down.
into Eqs. (30) and (31). This makes the system of equations well behaved. Now one must integrate these two additional equations in conjunction with Eq. (7).

One can also conduct “free” maneuvers (small amplitude) by removing the spring and dampers on the system.

III.H. Non-zero Thickness and Camber Correction

To correct the neglected effect of thickness and camber on the lift and moment, we introduce corrections to the lift coefficient and moment coefficients in Eqs. (21) and (22). It is believed that the corrections are static and do not depend on the angle of attack in the range of operation or the unsteady maneuvering. Thus we simply add the the zero angle of attack lift and moment coefficient to obtain

$$\tilde{C}_L = C_{L,0} + C_L$$
$$\tilde{C}_M = C_{M,0} + C_M$$

For example, on a clean NACA 4415 airfoil, the moment coefficient is nearly constant through the quarter chord for the range of angle of attacks in this study and thus $C_{M,0} \approx -0.118$. For the lift coefficient, the change in lift per angle of attack for the NACA 4415 is nearly that of a flat plate and thus we correct the lift by offsetting the lift coefficient by the zero angle of attack lift for a NACA 4415, i.e. $C_{L,0} \approx 0.4$. Of course, this correction will be wing section dependent and will be taken from experimental results.

IV. Results and Validation

IV.A. Experimental Moment and Lift Estimation

Accurate estimation of instantaneous aerodynamic pitching moment and lift force during dynamic maneuvers is necessary for generating validation data for modeling and simulation efforts. Steady state values can be measured with high accuracy by using the torque motor for the pitching moment and load cells for the lift force. The torque motor can produce a maximum torque of 30 N.m. In calibration tests performed by applying known torque loads through hanging known masses at a known moment arm showed a linear relation between the command to the motor and torque output. The standard deviation between the test data and the best linear fit was computed to be 0.04 N.m, which is equal to 0.13 % of the peak torque and 0.5 % of the pitching moment of the test model at 30 m/s. Aerodynamic moment is estimated by using the dynamics equation

$$I\ddot{\theta} = M(\alpha, \dot{\alpha}) + T$$

where $I$ is the moment of inertia of the model around the pitch axis, $M$ is the aerodynamic moment, and $T = Ku$ is the motor torque with $K$ and $u$ being the motor torque gain and command to the motor respectively. Losses in the axis bearings due to mechanical friction and damping are shown to be small compared to the terms in the above equation and hence they are ignored. At steady state, the left hand side of the above equation is zero, allowing the pitching moment to be estimated with less than 1 % error. When the model is in motion, the inertial term needs to be accounted for to get an accurate estimation of the pitching moment. Moment of inertia is identified experimentally with less than 10 % error. Angular acceleration is measured with the angular accelerometer, whose accuracy reduces with the increasing frequency of the motion due to its dynamic characteristics. Detailed analysis of the dynamic properties of the angular accelerometer will be presented in Muse et. al. (2008). For the present study, acceleration measurements are used directly to account for the inertial term for aerodynamic pitching moment estimation. In our future studies, efforts will be made to include a dynamic model for the accelerometer to enhance unsteady pitching moment estimates. In addition to uncertainty in the inertia estimate and dynamic characteristics of the accelerometer, noise in the acceleration measurements due to mechanical vibrations and electrical noise is another factor that degrades aerodynamic moment estimates.

Estimation of the aerodynamic lift force is analogous to moment estimation. Plunge dynamics equation used for this purpose is
\[ m\ddot{z} = L + F - mg \]

Lift at steady state is directly measured with the load cells within their accuracy, which was found through calibration to be less than 0.1 %. Correction for the inertial forces to estimate unsteady lift force is more complicated due to the spring sets on either side of the model. Springs on both sides have different spring constants and load masses leading to different natural frequencies for both sides. This causes asynchronous oscillations of the two sides resulting in rolling of the model within the spring deflection limits (< 1.3\(^\circ\)). Because of the rolling motion, unsteady aerodynamic lift force cannot be accurately estimated using the above point mass model. Validation data used in this study ignores the inertial terms in the lift equation. A 3D model to enhance aerodynamic lift estimations is under development.

IV.B. Validation with Experiments

Here we validate our results with experiments to gain confidence in the use of the low-order model for closed-loop control. For the tests a NACA 4415 is sinusoidal pitched very near quarter chord at a prescribed frequency in the presence of a free-stream flow. Experiments are run near a Reynolds number of \( Re_c \approx 8.5 \times 10^5 \) with a tunnel speed of 30 m/s, chord length of 0.4572 m. We define a reduced frequency,

\[ k_p = \frac{2\pi f_p c}{U} \]  

(32)

where \( f_p \) is the physical pitching frequency. As seen in figure 5, we see that our model predicts the correct lift and moment behavior for this relatively low reduced frequency of \( k_p = 0.068 \). Notice that even though there is a substantial amount of noise in \( \dot{\theta} \) the model still remains stable and bounded. It is this behavior that leads to think that the model will be robust in control situations. Note that the moment plotted in figure 5 is noisy due to the outer control loop which attempts to command the airfoil via an attached torque motor. If one were to filter the moment data from experiment, there would be good agreement between the model and the experiment.

We investigate an higher reduced frequencies by spanning several frequencies in a single run. The results are plotted in figure 6. Here, the filtered data is used for clarity (the noise level is roughly the same as that seen in figure 5). For these higher reduced frequencies, our model begins to depart from the experiments. There is an apparent phase shift in the moment data although the lift stays in phase with the experimental data.
The amplitudes given in the model also begin to depart from the experiment. Although it appears as though the model does not adequately capture the response for higher frequencies the results from comparisons to numerical results given in the next section suggests otherwise.

IV.C. Validation with Numerical Simulations

Here we employ the use of numerical simulations to validate the models in situations not realizable by the experiments. The numerical simulations are computed at the University of Texas at Austin using a Delayed Detached Eddy Simulation (DDES). The DDES scheme is a hybrid non-zonal Reynolds-Averaged Navier-Stokes (RANS) and Large-Eddy Simulation (LES) turbulence model based on the Detached Eddy Simulation (DES) model. Simulations are run at a free-stream Reynolds number, \( Re = 9 \times 10^5 \). For more information see Lopez (2008).

In figure 7 we show the impulsive start nature of the model versus DDES simulations. The model seems to do very well in capturing the response of the curve although we have added the correction defined in section III.H. Notice as \( t \) approaches larger times, the slope of the lift curve of both the model and simulation agree. The simulations also seem to lead to validate our assumption presented in section III.H that \( C_{L,0} \) is
constant even in unsteady responses.

A comparison to numerical simulations can study the model accuracy at frequencies several orders of magnitude higher than can be attained in experiments. In Figure 8, we compare the model undergoing prescribed sinusoidal pitching motion in the presence of a free-stream. The NACA 4415 airfoil pitches with the angle of attack prescribed by

\[ \theta(t) = A\left(1 - \cos\left(k_p t\right)\right) \]

where the reduced frequency is previously defined in Eq. (32). The simulations are run for 20 convective time units at zero angle of attack to allow the transients to die out. Once transients are adequately removed, the simulation moves the airfoil according to the prescribed path. As presented in Figure 8b, the model captures the correct characteristics for the gross behavior of the lift and moment. Both the magnitudes and phases are in accordance with the simulations. As expected, the model is not able to capture the higher frequency oscillations in the lift and moment due to the shedding of von Karman vortices from the non-sharp trailing-edge tip. To capture this effect, one must develop a higher-order model.

![Figure 8. Comparison between the model and DDES simulation of a NACA 4415 undergoing sinusoidal pitch. (a) Input angle of attack, pitch rate, and angular acceleration for the model and simulation. Pitching location is at the quarter chord (i.e. \(a = c/4\)) and pitch rate is at a reduced frequency of \(k_p = \frac{2\pi}{5} = 1.2566\). (b) Coefficient lift and moment. Moment is taken at quarter chord. (—) Model, (—) Simulation.](image)

### IV.D. Example: Vibration Suppression

It has been shown that given certain conditions, an airfoil will undergo non-stalled vortex-induced vibration when the flow is attached. Here we simulate a specific case of non-stalled vibration. The parameters are chosen to give a rather oscillatory response (see Figure 9, blue). The reduced frequency of the oscillations are rather high still and the vibrations do not damp out until a few convective time units. In an attempt to suppress the vibration we apply a simple control law such that,

\[ u = -K_G\dot{\theta}. \]  

The results of a simple control strategy can be seen in Figure 9. The oscillatory behavior seen in all rigid body states is now reduced substantially. It is shown that one can use the control vortex to actively suppress the vibration phenomenon discussed here.

### IV.E. Non-linearity

Because a vortex is created at each time the sign of the nascent vortex strength changes the model constantly resets the fluid dynamic states, i.e. it sets location of the vortex at \(\xi_c = c/2\) and the corresponding strength to \(\Gamma_c = 0\). This is a result of the low-order nature of the model. In this model we attempt to keep the number of model states fixed, but in actuality the number of vortices increases over the maneuvering

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Figure 9. An example of non-stall vortex-induced vibration. The parameters (see figure 4) are $m = 1.0$, $I = 1.0$, $k_y = 100.0$, $k_\theta = 100.0$, and $b_y = b_\theta = 0$. (—) Uncontrolled model, (—) Controlled model.

Figure 10. Example of the non-linearity present in this dynamical system. This is the data gotten from the vibration simulation presented in figure 9.
time. To circumvent this problem the previous states are reused for the newest vortex. This causes a discontinuity in both the vortex position and strength when a new vortex is shed. This behavior can be seen in figure 10. We are unsure whether it will be difficult to design a control law for a non-linearity of this kind because it is hard to characterize.

V. Conclusion

A novel low-order model is developed to determine the aerodynamic characteristics of an arbitrary airfoil undergoing pitching and plunging flight without leading edge separation. The fluid dynamic model is based on the transport of vorticity and conservation of circulation. Various assumptions are made to develop a model such that it is tractable for adaptive closed-loop control. We reduce the number of states to the bare minimum with the outcome being a model with two distinct fluid dynamic states that track the location and strength of the nearest trailing edge vortex. This results in a singular and extremely non-linear differential equation for the vortex location and an algebraic constraint for the vortex strength. The singularity is present at the event when a new vortex is formed and a special change of variables is used to remove the singularity. The extreme non-linearity arises due to the vortex creation criteria; a vortex cannot lose strength. The fluid dynamic states which tracks the nascent vortex, is constantly reset to the initial conditions of a new vortex.

The aerodynamic force and moment are derived from a relation for the fluid dynamic impulse and consist of an added mass term, a quasisteady lift term, and a term due to the unsteadiness of the wake vorticity. The added mass forces are found to be in accordance to the accepted values. A correction for bodies with thickness and camber is proposed and simulations seem to suggest the correction is valid.

A control vortex is introduced into the unsteady low-order model to represent the trapped vorticity effect of both a SS and PS synthetic jet near the trailing edge. This is adequate for this specific case where one is interested in modifying the aerodynamic performance in the attached flow regime via virtual aerodynamic shaping. The control vortex is modeled to sit near the location of the actuation with a varying strength. Based on the assumption that the formation time scale of the trapped vorticity is much smaller than the convective time scale, a relationship between control voltage and vortex strength is experimentally determined from steady data. The data strength of the control vortex also showed a slight dependence on angle of attack.

Rigid body dynamics are also added to the low-order model to completely couple the fluid and structure interaction. This results in a complete closed set of equations for the rigid body states and the fluid states. An example of a fluid-structure interaction problem where the attached flow generates trailing edge vortices that cause vibration in the structure is presented. A simple control strategy is introduced to alleviate vibration in the model.

The model in the absence of control is validated with experimental data and numerical simulations. For both cases, a NACA 4415 airfoil is prescribed a sinusoidal oscillation in pitch at a specific reduced frequency. In experiments, reduced frequencies up to $k_p \approx 0.2$ are realizable. In the lower range of reduced frequencies ($k_p < 0.1$) the low-order model and the experiments agree remarkably well. In addition, the low-order model is robust to noisy input data and provided smooth lift and moment responses. In the higher reduced frequency range ($k_p > 0.1$) the low-order model began to deviate from the experiments. There appears to be a phase shift in the moment comparison in addition to a discrepancy in amplitude for both the lift and moment, and thus the model seems to break down for more dynamic maneuvers. In comparisons to numerical simulations, results seem to suggest otherwise. At nearly an order of magnitude higher reduced frequencies, the comparison of the low-order model to numerical simulations demonstrates a reasonable agreement in both phase and amplitude of lift and moment coefficients. As expected, the low-order model is not able to capture the oscillations in lift and moment due to the small scale von Kármán vortices.

The future work now revolves around further correcting the low-order model to more accurately describe the flow physics. One major area of research is to find a more valid characterization of the control vortex versus the control voltage input. In addition a corrective term may be needed for the convection of the nascent vortex due to the presence of the control vortex since it has been seen experimentally that a synthetic jet deflects the wake vortices. This may be viewed as an artificial adjustment of the well-known Kutta condition.
In conjunction with this work, the low-order model will soon be implemented in the closed-loop adaptive control environment.

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References